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(NASA-CR-120488) THE DESIGN AND ANALYSIS OF AN AUTOMATIC FREQUENCY CONTROL SYSTEM (Auburn Univ.) 63 p

N74-77382

Unclas
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ENGINEERING EXPERIMENT STATION



AUBURN UNIVERSITY

AUBURN, ALABAMA



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TECHNICAL REPORT

# THE DESIGN AND ANALYSIS OF AN AUTOMATIC FREQUENCY CONTROL SYSTEM

Prepared by

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August 22, 1968

CONTRACT NAS8-20765

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## FOREWORD

This is a special technical report of a study conducted by the Electrical Engineering Department, Auburn University, in cooperation with Project NAS8-20765, concerned with the automatic frequency control of telemetry transmitting. Mr. Larry Rosser, who conducted this research, was supported by fellowship funds.

## ABSTRACT

This study is concerned with the design and analysis of an automatic frequency control system utilizing a voltage-controlled Colpitts oscillator and a Foster-Seeley discriminator analyzed by methods common to feedback control. From this analysis, criteria which may be useful in the design of a physical system are found.

The design criteria for the circuit of the automatic frequency control system are presented, and the accuracy of the system is shown to depend upon the value of the open-loop gain of the system, the frequency stability of individual components in the system, and the characteristics of a low-pass filter included in the feedback path of the system.

A model of the system analyzed, operating at a frequency of the 440 kHz for convenience, was constructed. The results of the several tests performed on the model are presented.

The effects of individual components on the system are studied and methods of selecting the components for a particular system are given. It is shown that the closed-loop frequency stability of the oscillator may exceed the

stability of the reference frequency of the discriminator in some cases. This is confirmed by a test performed which shows that by using a discriminator with a center frequency stability of .1 percent and an oscillator with an open-loop stability of 1.5 percent over a temperature range of 5°C to 70°C, the overall frequency stability of the system can exceed .05 percent over the same temperature range. This was accomplished by making the slopes of the frequency versus temperature curves of the oscillator and the discriminator of opposite sense.

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# THE DESIGN AND ANALYSIS OF AN AUTOMATIC FREQUENCY CONTROL SYSTEM

L. A. Rosser and M. A. Honnell

## I. INTRODUCTION

Several different types of automatic frequency control systems are commonly employed. In some a constant frequency difference is maintained between a reference frequency and the controlled frequency. In others an attempt is made to maintain zero frequency difference between the reference frequency and the controlled frequency. The analysis presented here is concerned with the latter.

If frequency stability is the only requirement, then there are no great difficulties encountered in the design of a stable oscillator. Oscillators utilizing the mechanical resonance of a crystal to determine frequency can be built with a frequency stability of $1X10^{-6}\%$ [1]. However, the same factors which give the crystal oscillator its stability also limit its capability for direct frequency modulation. It should be realized that the requirement that an oscillator cover a range of frequencies imposes difficulties not present for fixed frequency oscillators [2].

The need for a method of frequency stabilization of frequency modulated oscillators was recognized long ago [3], and the design of a system which would accomplish this purpose followed soon after [4]. There are many methods now available for stabilizing the frequency of oscillation of frequency modulation transmission systems which use an oscillator with the capability of swinging over a range of frequencies. Perhaps the oldest and the simplest is the discriminator automatic frequency control system. Its simplicity and economical construction are factors which make this system more desirable than others.

Automatic frequency control (AFC) is a closed-loop regulating system which automatically adjusts an oscillator to maintain a constant average frequency at the output of the device. The average frequency will remain constant but the instantaneous frequency will vary if information is applied through frequency modulation.

There are two basic requirements for any automatic frequency control system. It must have a frequency-controllable oscillator and some form of frequency sensitive detector whose output is fed to an oscillator frequency control circuit.

In an automatic frequency control system the output of an oscillator is fed into a circuit whose output is a function of the difference between the frequency of the oscillator and the reference frequency of the circuit.

The output of the detecting circuit is then applied to the oscillator frequency control circuit in such a manner as to reduce the frequency difference, thus reducing the error in the oscillator frequency. This system requires a small, but finite, error of the controlled variable (the output frequency) in order to operate. Systems avoiding this steady state error have been designed [5].

## II. THEORETICAL ANALYSIS

Figure 1 shows the basic components of an elementary automatic frequency control system. There are many variations of the basic AFC system, but the analysis presented here will be limited to the system shown in Figure 1. The system contains the following: (1) an oscillator with a nominal frequency equal to the desired output frequency; (2) some form of variable reactance or other means for voltage control of the frequency of the oscillator; (3) a frequency discriminator which detects any difference in the output frequency and the desired output frequency; and (4) a low-pass filter which filters the output voltage of the frequency discriminator to remove any modulation components before it is applied to the variable reactance of the oscillator. In the analysis of the system shown in Figure 1, it will be assumed that additional amplifiers can be used to provide the desired value of loop gain.

Qualitatively, the operation of the system can be understood by realizing that the discriminator produces a dc output voltage which is proportional to the difference in frequency of the output of the oscillator and the center

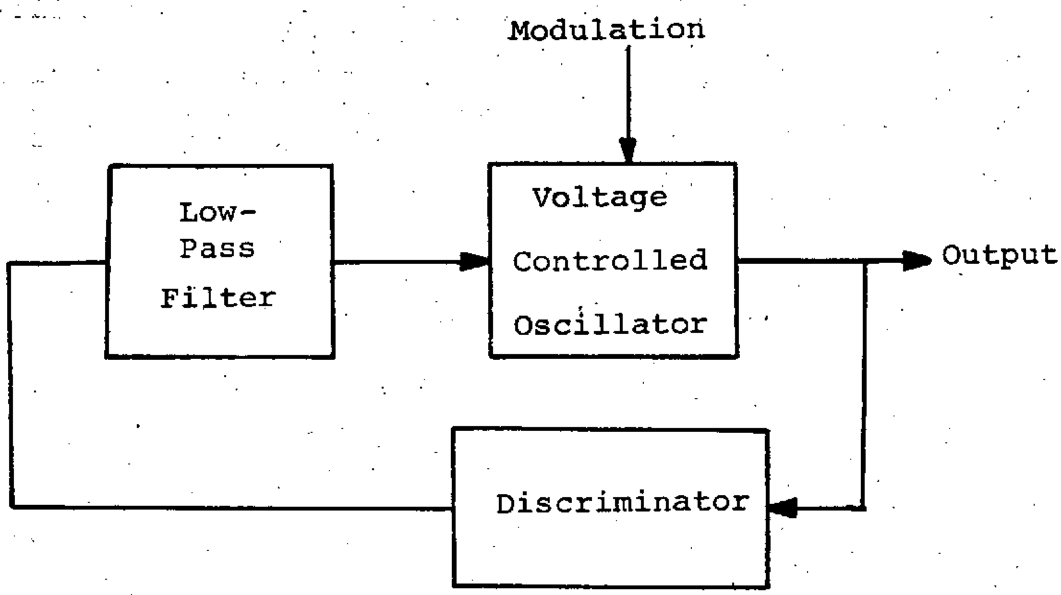



FIGURE 1.--A basic automatic frequency control system

frequency of the discriminator. This voltage is applied to the variable reactance in the oscillator circuit in such a manner that the frequency of the oscillator is pulled closer to the center frequency of the discriminator. Since the direct voltage output of the discriminator is caused by the difference between the frequency of the oscillator and the center frequency of the discriminator, it is obvious that the frequency error cannot be reduced to zero. However, it has been shown that this error can be made very small [6].

In the system analysis the discriminator characteristic and the change in oscillator frequency caused by the variable reactance tuning the oscillator will be assumed to be linear. Practically these assumptions are valid only over a given region of operation. The automatic frequency control cannot be expected to stabilize a signal which lies outside the range of operation of the discriminator. Usually poorer stability results as the range of the discriminator is increased.

There are certain properties of the automatic frequency control system which are of great interest. The more important of these include the capture range, the hold-in range, and the filter bandwidth. These properties will be discussed throughout the analysis and design of the AFC system.

Shown in Figure 2(a) and 2(b) are mathematical models of the AFC system that will be used in the analysis. In Figure 2(a) the gain of the discriminator in volts per Hz is denoted by $K_1$. $K_2$ represents the gain of the voltage-controlled oscillator in Hz per volt. G(s) is the frequency-dependent part of the open-loop transfer function of which the most significant term is the low-pass filter transfer function. $K_3$ is dimensionless and includes the gain of any amplifiers and summing junctions. The output frequency, $f_o$, is the sum of the uncompensated voltage-controlled oscillator (VCO) frequency, $f_a$, and a compensation term, $f_c$, obtained by passing the output of the oscillator through the discriminator with a center frequency of $f_d$.

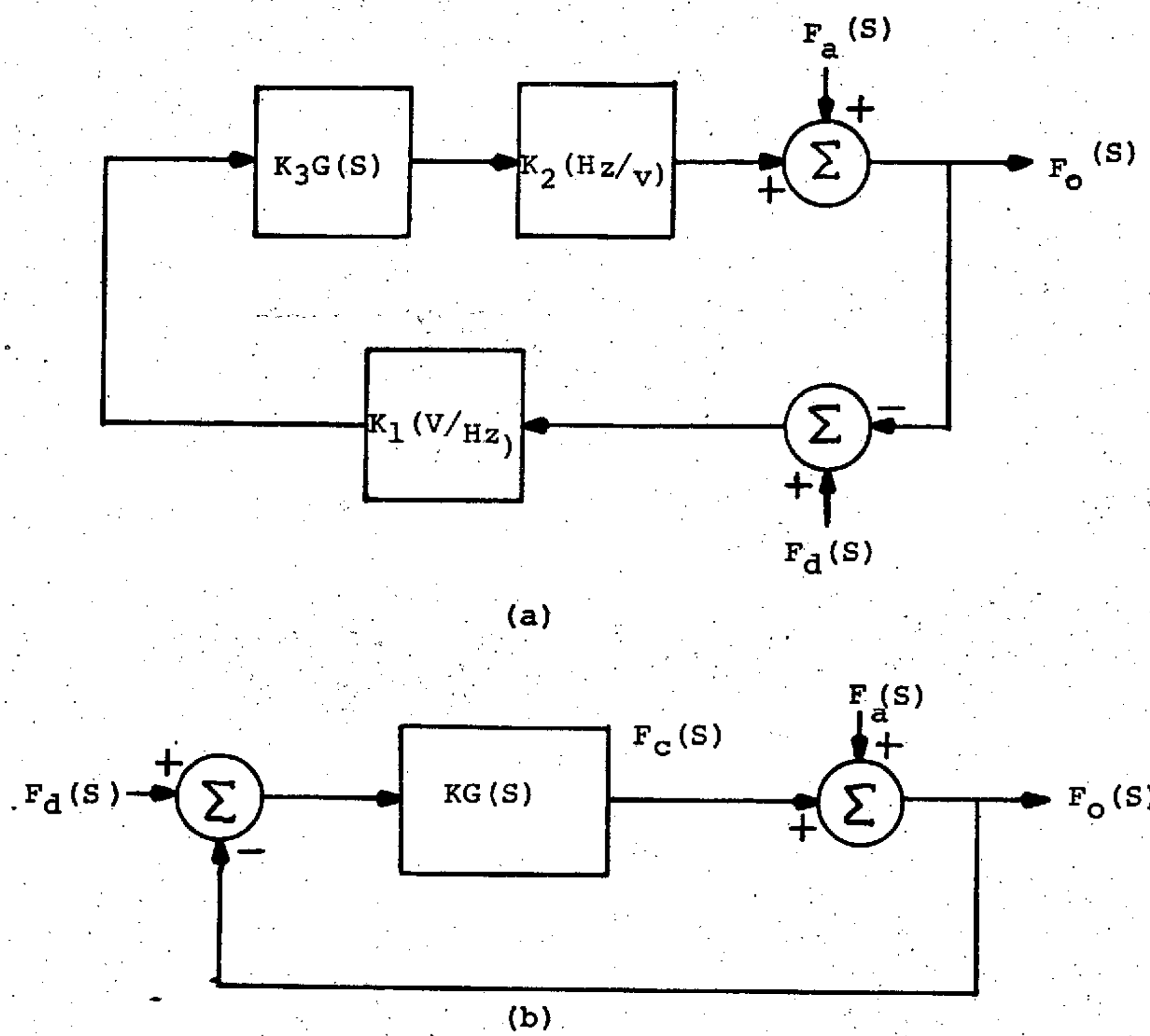
The closed-loop response for the equivalent system shown in Figure 2(b) is given by:

$$F_o = \frac{F_a + K\,G(S)\,F_d}{1 + K\,G(S)} \tag{1}$$

where K is a dimensionless gain constant obtained by combining the various gains.

It is necessary to pick a particular value for G(S) if any conclusions are to be drawn from (1). If the low-pass filter shown in Figure 3 is used, the transfer function is given by:

$$G(S) = \frac{1}{RCS + 1} \tag{2}$$




Note: $F_a(S)$ is the Laplace transform of $f_a(t)$

$F_d(S)$ is the Laplace transform of $f_d(t)$

$F_c(S)$ is the Laplace transform of $f_c(t)$

FIGURE 2.--(a) Mathematical model of AFC loop; (b) Equivalent simplified model of AFC loop

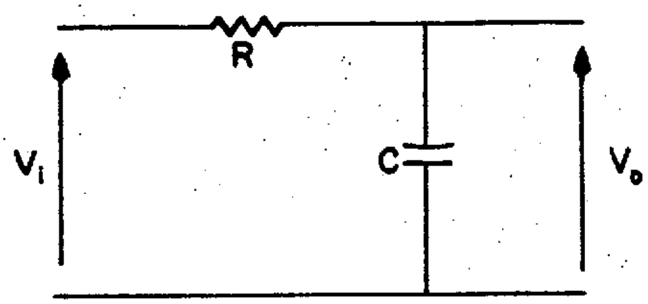



FIGURE 3.--An RC low-pass filter

Using this G(S) in (1) and rearranging terms yields the following closed-loop response

$$F_o = F_d + \frac{F_a - F_d}{1 + K/(RCS + 1)} \qquad (3)$$

Employing the final value theorem [7], and noting that $f_a$ and $f_d$ are constants, gives the steady-state solution

$$f_o = f_d + \frac{f_a - f_d}{1 + K} \qquad (4)$$

Equation (4) shows that the output frequency differs from the reference frequency $f_d$ by an amount equal to the deviation of the uncompensated VCO frequency from the discriminator center frequency divided by one plus the open-loop gain. If the steady-state error is to approach zero, a very high loop gain is required.

The difference between the actual output frequency and the desired frequency is given by [8]

$$\Delta f_o = \frac{\Delta f_a}{1 + K} + \frac{K \Delta f_d}{1 + K} \qquad (5)$$

where

$$\Delta f_o = f_o - \bar{f}_o$$

$$\Delta f_a = f_a - \bar{f}_a$$

$$\Delta f_d = f_d - \bar{f}_d$$

and the "barred" quantities represent the free running or designed values. If the frequency stability, $\delta$, is defined as

$$\delta = \frac{\Delta f}{f} \times 100\ \% \tag{6}$$

then from (5), the output stability is

$$\delta_o = \frac{1}{1+K}\left(1 - \frac{f_c}{f_o}\right)\delta_a + \frac{K}{1+K}\frac{f_d}{f_o}\delta_d \tag{7}$$

where $\delta_o$ is the output stability in percent, $\delta_a$ is the voltage-controlled oscillator stability in percent, and $\delta_d$ is the discriminator stability in percent. Note that $\delta_o$ is dependent upon the frequency error in the system.

Once the loop has control, the error in frequency will be small enough to make the approximations

$$f_c/f_o \approx 0$$

$$f_d/f_o \approx 1$$

if the open-loop gain is made large enough. With these approximations and rearrangement of (7) the output stability can be approximated as

$$\delta_o = \delta_d + \frac{\delta_a - \delta_d}{1+K} \tag{8}$$

When $\delta_a$ and $\delta_d$ have the same sign, the stability of the output frequency can, at best, be no better than the stability of the center frequency of the discriminator.

Typical values of $\delta_a$ and $\delta_d$ can be found to illustrate how (8) can be used to determine the value of open-loop gain required for a given output stability. A typical value of $\delta_a$ of two percent will cover all changes in VCO frequencies, including initial manufacturing tolerances, variations due to temperature as well as aging effects [9]. A typical value of $\delta_d$ = .05 percent will cover all changes in the center frequency of the discriminator due to temperature variations and aging. A discriminator with a center frequency stability of $\delta_d < .05\%$ was constructed; therefore .05 percent is a realistic value. If a stability of .06 percent is desired in the output frequency, then from (8) an open loop gain, K, of 194 or about 45.7 db is required.

If an inductor is added in the series arm of the filter shown in Figure 3, the behavior of AFC loop will be altered. The filter with the inductor is shown in Figure 4.

The transfer function associated with the filter in Figure 4 is

$$G(S) = \frac{1}{S^2LC + RCS+1} \tag{9}$$

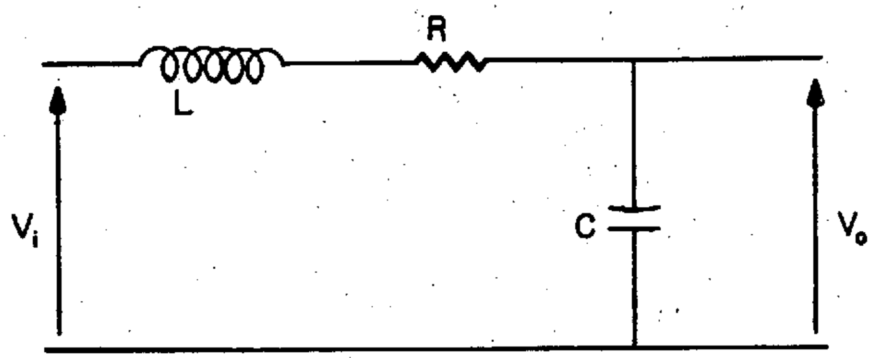



FIGURE 4.--**A** three-element low-pass filter

The closed-loop response, using this filter transfer function, is

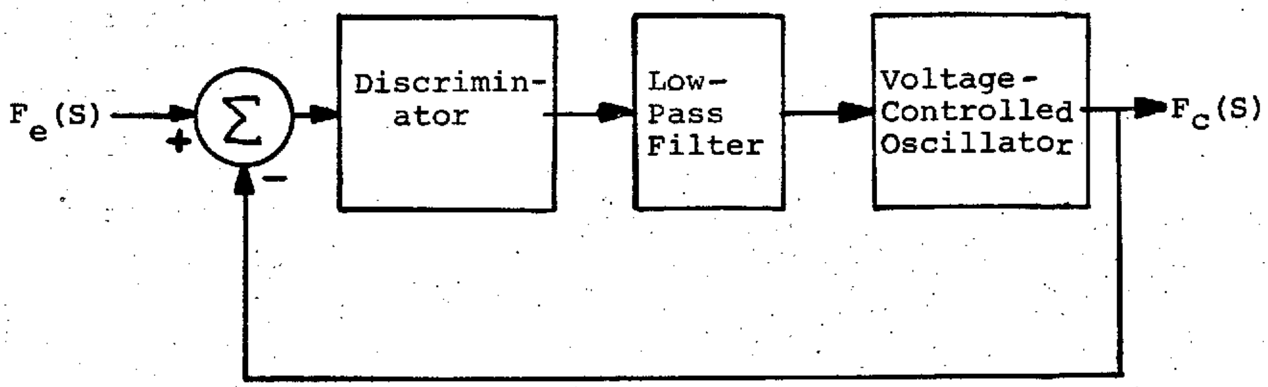
$$F_o = \frac{F_a + KF_d/(S^2LC+RCS+1)}{1+K/(S^2LC+RCS+1)} \tag{10}$$

Equation (10) may be rearranged to yield

$$F_o = \frac{F_a(S^2LC+RCS+1) + KF_d}{S^2LC+RCS+K+1} \tag{11}$$

Because the highest power of S in the denominator of the transfer function is two, this loop is known as a "second-order loop." The second-order loop is widely used because of its simplicity and good performance. If the final value theorem is used, then the result is the same as when the RC filter was used and is given in (4). With this filter a high gain is required to hold the output frequency to the reference frequency. If the same methods of analysis that were used to analyze the RC filter are used here, it can be shown that the output stability is related to the stabilities of the individual components as previously given in (8).

The system may be analyzed with the error frequency as the input and the correction frequency as the output for added insight into the performance of the system. Shown in Figure 5 is the AFC loop with the configuration




Note: $F_e(S)$ is the Laplace transform of $f_e(t)$

$F_c(S)$ is the Laplace transform of $f_c(t)$

FIGURE 5.--A block diagram of an AFC system

mentioned above.

In Figure 5 the input, $f_e$, is the frequency error of the system. The output, $f_c$, is taken to be the correction term which the system produces. It will be of interest to see how closely the output can follow the input for this arrangement. The actual output frequency is not observable in this arrangement but is given by

$$f_o = fa + f_e - f_c \tag{11}$$

where

$f_o$ = output frequency

$f_a$ = free-running frequency of oscillator

$f_e$ = frequency error term

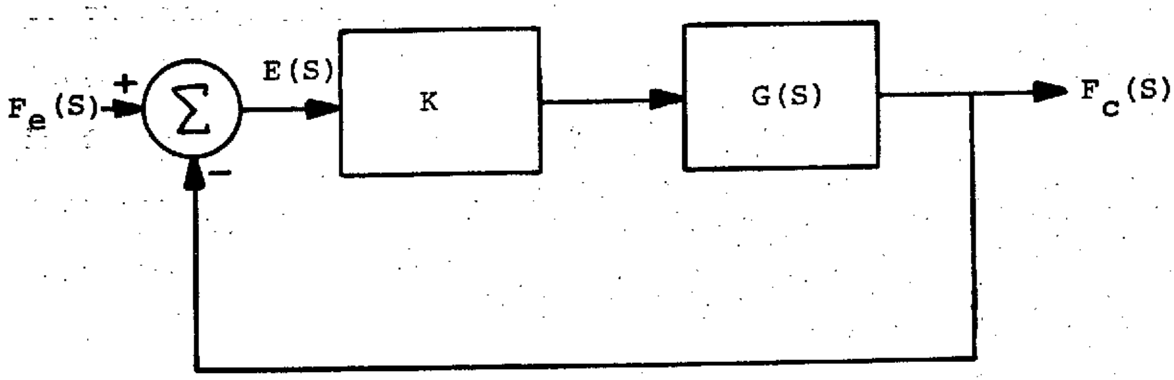
$f_c$ = correction term

For a constant output frequency, $f_c$ should follow $f_e$ very closely.

A simplified model of the system in Figure 5 is shown in Figure 6. The closed-loop response of the system in Figure 6 is

$$F_c = \frac{K\ G(S)}{1+KG(S)} \quad F_e \tag{12}$$

The error or actuating signal, E, is related to the input by




Note: $F_c(S)$ is the Laplace transform of $f_c(t)$

$F_e(S)$ is the Laplace transform of $f_e(t)$

$E(S)$ is the Laplace transform of the error signal, $e(t)$

FIGURE 6.--Simplified mathematical model of system shown in Figure 5

$$E = \frac{1}{1+KG(S)} \quad F_e \tag{13}$$

If the transfer function, G(S), for the filter in Figure 3 is used in (12), then the system is analogous to the non-integrating system found in servomechanisms [10] and will have a steady-state error to a step function in frequency. It must be emphasized that the AFC loop must have a finite frequency error to function. This error may be made very small but cannot be eliminated.

If the filter of Figure 3 is used in the feedback path, the loop transfer function may be written as

$$H(S) = \frac{F_c}{F_e} = \frac{1}{\tau S+1+K} \tag{14}$$

where $\tau$ =RC is the time constant of the filter. There are two parameters which can be varied, $\tau$ and the open-loop gain, K. Since there are only two quantities to consider in the design, the bandwidth of the filter, and the gain, they may be chosen completely independently of each other.

If the filter shown in Figure 4 is used, the closed-loop response is

$$F_c = \frac{K}{LCS^2 + RCS + 1 + K} \quad F_e \tag{15}$$

Equation (15) may be rearranged to yield

$$\frac{F_c}{F_e} = \frac{A}{S^2 + 2\zeta\omega_n S + \omega_n^2} \qquad (16)$$

where

$$A = K/LC$$

$$\omega_n = \sqrt{\frac{1+K}{LC}}$$

$$\zeta = \frac{R}{2}\sqrt{\frac{C}{L(1+K)}}$$

The parameter $\omega_n$ is the undamped natural frequency, and $\zeta$ is the damping ratio. The response of the system depends upon the values of $\omega_n$ and $\zeta$. The amount of damping ($\zeta$) determines the oscillatory nature of the response, and $\omega_n$ affects the rise time. The frequency response for several values of damping factor ($\zeta$) is plotted in Figure 7 [11].

It can be seen that the feedback loop performs the low-pass filtering operation which is necessary in a frequency-modulated system. The feedback loop must not compensate for variations in frequency due to modulation but must compensate only the slower variations due to random effects. The bandwidth of the filter can be chosen so that only the variations due to random effects are compensated. The response of the entire system will be determined by the values chosen for $\zeta$, $\omega_n$, and A.

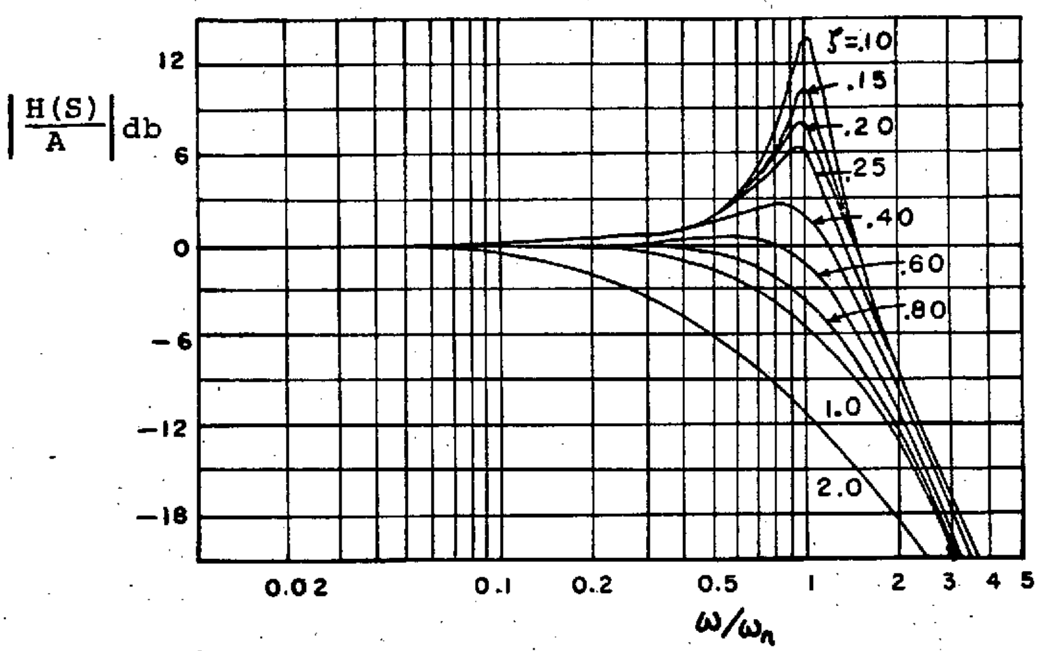



FIGURE 7.--The frequency response of a second-order system for several values of $\zeta$

There are four circuit parameters available (K, L, C, and R), and there are three loop parameters to be determined ($\omega_n$, $\zeta$ , and the open-loop gain). Therefore the loop parameters can be chosen independently of each other. A high gain can be chosen for a low error; values for L and C can then be chosen to give the best natural frequency. R is then available to determine the damping ratio independently of the gain and undamped natural frequency.

If the oscillator and discriminator were not limited to some region of operation, the system would capture and hold to the center frequency of the discriminator any frequency at which the voltage-controlled oscillator oscillates. Practically, there are physical limitations which are discussed in the design section.

## III. DESIGN CONSIDERATIONS

There may be many requirements placed on a voltage-controlled oscillator. These requirements depend upon the application for which the oscillator is used. Some of the more important requirements are:

(1) Large tuning range.

(2) Reasonable rate of change in frequency with respect to a change in voltage.

(3) Capability for accepting wide-band frequency modulation.

These requirements usually conflict with each other and for this reason a compromise in the design of the VCO must be reached.

The most common types of voltage-controlled oscillators in use are: crystal oscillators, LC oscillators, and RC multivibrators. Of these the crystal oscillator (VCXO) is the most stable, and the RC multivibrator is the least stable. When high-Q crystals are used in the VCXO, for greatest stability, the frequency deviation range is small. Crystal parameters are also temperature sensitive to some extent. Temperature changes and fluctuations need to be avoided.

If a wider deviation range is required, stability must be sacrificed. An LC oscillator having a wider deviation range can still maintain fair stability of frequency. The Hartley, Colpitts, and Clapp circuits are commonly used for this purpose. Although saturable inductors have been used for the tuning element, the varactor can give good results if used for this purpose. A varactor is a semiconductor junction diode having a capacitance proportional to the reverse bias across its terminals. There are two major limitations to using a varactor for voltage tuning. These limitations are due to:

(1) degradation in the circuit quality factor, Q,

(2) the finite tuning range implied by the maximum capacitance of the diode.

However, the convenience of the varactor often offsets these disadvantages.

If a large tuning range is much more important than stability, then relaxation oscillators such as multivibrators and blocking oscillators can be used at a lower cost. The operating frequencies of these oscillators have been limited to a few megacycles, but the linearity of frequency versus control voltage is usually very good.

The analysis of the AFC system has been based on the assumption that the discriminator is a linear device.

This assumption, which is good for small errors, is completely invalidated as the error increases. Figure 8 [12] shows the relationship between the input frequency and the output voltage of a typical discriminator. The distance between the two maxima is often called the peak separation or, sometimes, the bandwidth of the discriminator. Over the range of frequencies included in the bandwidth, the discriminator will provide a dc voltage whose amplitude is approximately linearly related to the frequency deviation. The slope of the line in Figure 8 is a measure of the sensitivity of the discriminator and is greatest at the center frequency. The actual pull-in range, or capture range, of the system will depend upon the oscillator, the amplifier characteristics and the characteristics of the discriminator. In some cases the bandwidth of the discriminator may be the limiting factor in determining the range of frequencies which may be compensated. If this is true and if the loop has sufficiently high gain, any frequency deviation can be corrected if the output of the discriminator can be detected. Here "detected" implies that the magnitude of the output signal exceeds that of any random perturbations in the system. When the signal becomes too small, the offset voltage of a dc amplifier, if one is used, quite often obscures it.

Another quantity of interest is the hold-in range of

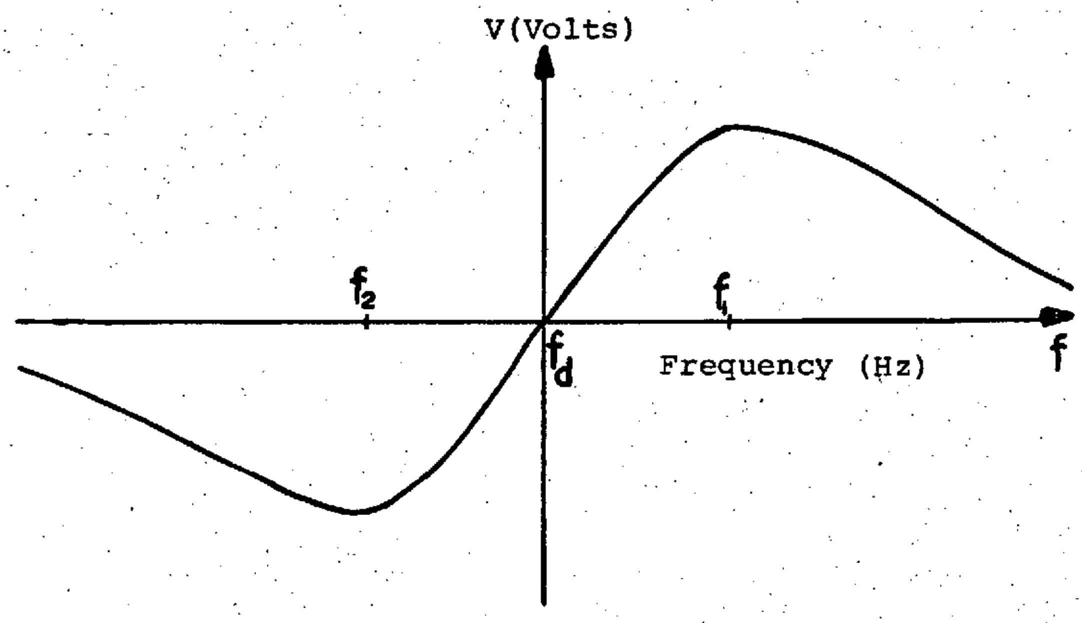



FIGURE 8.--Typical characteristic curve of a discriminator

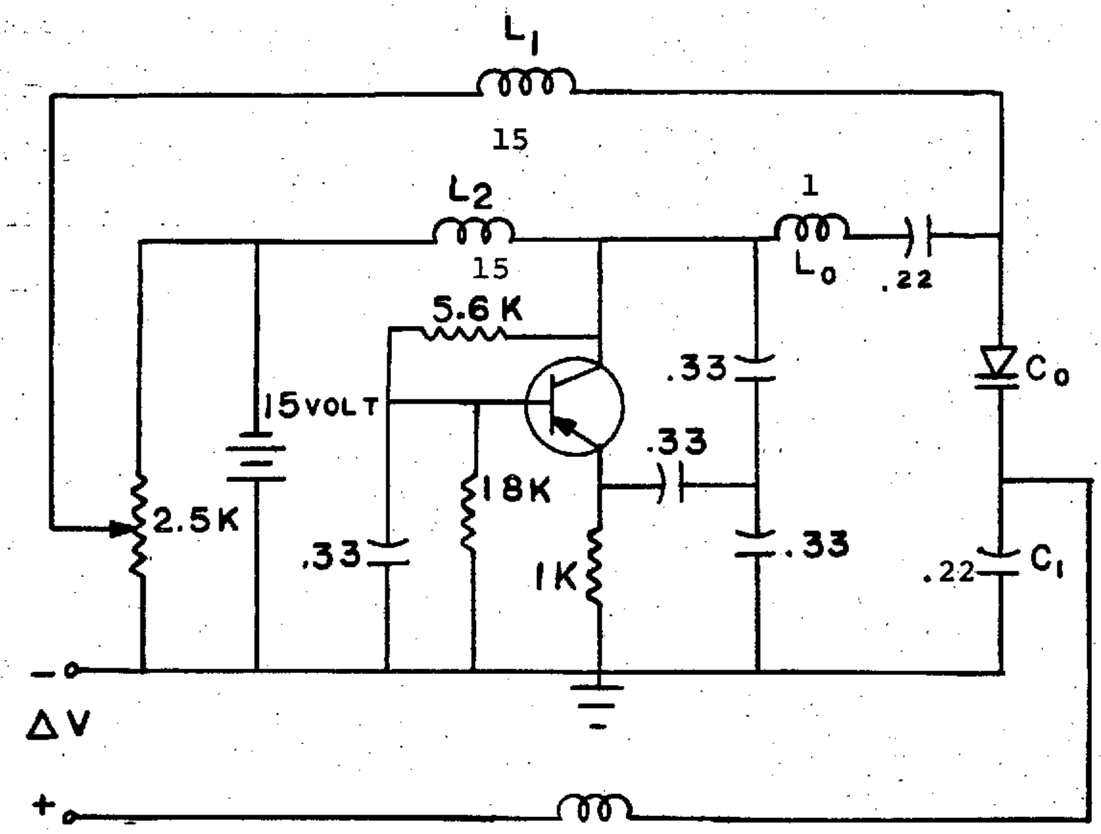
the loop. This is a measure of how well the feedback loop will hold the oscillator frequency to the desired frequency once the feedback loop has gained control over the system. If the system is entirely linear and the gain is sufficiently high, the output frequency will be held to $f_d$ regardless of the variation of parameters of the oscillator. However, as with all physical components, a saturation point, beyond which no amount of gain can correct the frequency, is reached. The loop amplifier has some maximum voltage that it can deliver and the varactor of the oscillator has some maximum voltage that it can accept. If either of these limits is exceeded, the loop loses control. The hold-in range, then, is determined by these quantities. If it is assumed the varactor tunes the oscillator over a range of frequencies much larger than the bandwidth of the discriminator, it is found that the hold-in range exceeds the pull-in range. This is due to the fact that the pull-in range is dependent upon the region of the discriminator curve where the magnitude of the correction voltage is very low whereas the hold-in range is practically independent of the exact nature of the discriminator curve if sufficient gain is used in the loop.

Usually a loop gain greater than the combined gain of the discriminator, oscillator, and filter is desired.

This gain may be provided by an amplifier which may introduce new problems such as offset voltages and drift. An offset voltage is the largest problem if a dc amplifier is used to amplify the error voltage from the discriminator. If integrated-circuit operational amplifiers are used, the offset voltage can be reduced to a minimum, and the desired gain may be obtained. However, for temperature variations, drift in the offset voltage can occur thereby limiting both the capture range and the hold-in range severely.

The characteristics of the low-pass filter are determined by the type of filter used. Proper values for the elements must be chosen so that the loop will not attempt to follow changes in frequency due to the lowest rate of frequency modulation. The filter must be carefully designed as the response of the entire loop depends upon its characteristics.

To demonstrate that a wide tuning band is compatible with good stability, a varactor-tuned LC oscillator has been constructed. The oscillator circuit, shown in Figure 9, employs a Motorola 2N4402 transistor in a Colpitts configuration with the Clapp modification. The varactor chosen to tune the circuit should have a range of capacitance which allows tuning of the oscillator over the desired frequency range. As mentioned previously,




Note: All values are in ohms, millihenries or microfarads unless otherwise indicated.

The transistor is a Motorola 2N4402.

FIGURE 9.--A varactor-tuned LC oscillator

the degradation of the quality factor, Q, of the tuned circuit must also be taken into consideration.

The voltage applied across the diode junction determines the capacitance of the varactor and, therefore, the resonant frequency of the tuned circuit. Since the varactor has some minimum capacitance at the breakdown voltage, there is a maximum resonant frequency. The varactor also has a maximum capacitance, which is determined by the highest forward voltage, $V_{min}$, that can be applied before forward conduction current and noise become intolerable. $C_{max}$ is usually much greater than $C_{min}$, and it determines the minimum resonant frequency. The ratio of maximum to minimum capacitance of the varactor is a good indication of the ability of the varactor to tune the oscillator over a wide frequency range since

$$\frac{\omega_{min}}{\omega_{max}} = \sqrt{\frac{C_{max}}{C_{min}}} \tag{17}$$

Aside from limitations on the capacitance of the diode, the tuning range may be restricted by the losses in the varactor. The quality factor, Q, of a tuned circuit is given by

$$Q = \frac{Q_L \, Q_C}{Q_L + Q_C} \tag{18}$$

where $Q_L$ is the Q of the inductor at the resonant frequency and $Q_c$ is the Q of the capacitance of the tuned circuit and is a function of the applied voltage, $V_o$. The Q of a varactor is given by

$$Q_c(V_o) \quad = \quad \frac{1}{\omega_r(V_o)R_sC(V_o)} \tag{19}$$

where $\omega_r$ is the resonant radian frequency, $\sqrt{\frac{1}{LC}}$ , and $R_s$ is the series resistance of the varactor. If (19) is rearranged and the definition of $\omega_r$ used, it is found that

$$Q_c(V_o) \quad = \quad \frac{\omega_{r(V_o)}\ L}{R_s} \tag{20}$$

Thus the Q of the varactor varies linearly with the resonant frequency. The maximum Q is obtained at the maximum resonant frequency which occurs when the applied voltage is near the breakdown voltage. In some cases there may be values of $Q_c$ below some specific value which are intolerable, in which case, the minimum resonant frequency would be that frequency at which the specified minimum $Q_c$ is reached. Therefore in some cases the minimum resonant frequency may be dictated by the maximum capacitance of the varactor, and in other cases by the minimum tolerable Q.

There may be additional restrictions on the varactor if the rf power level is large. The average capacitance, $C_o$, which determines the resonant frequency, depends upon the drive level as well as the bias voltage. Another restriction on the allowed variation of voltage $V_o$ also exists; it can no longer range from $V_{min}$ to the breakdown voltage $V_B$. A third restriction is that the rf power dissipated in the varactor may be too great, either because it exceeds the dissipation rating of the diode, or because the tank circuit efficiency is lowered.

For a sinusoidal voltage applied across the varactor, the average capacitance $C_o$ will be a function of the tuning voltage. However, it is also a function of the rf power level. Penfield has shown that the dissipated rf power of a varactor is related to the rf level by [13]

$$P = .281\ a^2\ P_{norm}\ \frac{\omega_r}{\omega_c} \tag{21}$$

where $\omega_r$ is the resonant frequency, $\omega_c$ is the cut-off frequency at the breakdown voltage, $P_{norm}$ is the varactor normalization power ( $\frac{V_B+V_{min}}{R_S}$), and "a" is the ratio of peak-to-peak rf charge to the maximum stored charge of the varactor. By this definition it is seen that "a" is restricted to lie between zero (no rf excitation) and one (full excitation).

For a specific power level the range of $V_o$ is restricted since $\omega_r$ is dependent upon $V_o$. Thus, as the power level increases, the tuning range decreases. This restriction on $V_o$ arises because of the requirement that the instantaneous voltage (the sum of the dc bias voltage and the instantaneous rf voltage) must lie between $V_{min}$ and $V_B$. If the rf-voltage swing is very large, then $V_o$ cannot move very far before either the positive or negative peaks exceed the possible voltage range.

This discussion has indicated that for the largest tuning range, a varactor with a high ratio of $C_{max}$ to $C_{min}$ should be used. In addition, extra capacitive energy storage should be avoided. If the varactor is in series with additional capacitors, both $C_{max}$ and $C_{min}$ will be decreased by this capacitance and the tuning range will suffer.

The varactor used in the circuit model was a parallel combination of five Microwave Associates, Inc. diodes, type MA4324F, each having the characteristic of (1) a breakdown voltage of -24 volts, (2) a nominal capacitance of 10-20 picofarads, and (3) a series resistance of less than 1.3 ohms.

The performance of varactors used for tuning is discussed in the literature [14].

To allow for a wider tuning range all capaciters in the tuned circuit have a much higher value of capacitance than does the diode.

The inductors $L_1$ and $L_2$ are radio-frequency chokes used to isolate the dc supply from the rf circuit of the oscillator. They present a high impedance at 440 kHz, which is the designed frequency of oscillation.

A modified version of the Foster-Seeley discriminator was used in the constructed model. A schematic diagram is shown in Figure 10. The feedthrough from the primary to the secondary is to a node splitting the capacitance instead of the center tap of the secondary coil as is often seen. The typical output of this discriminator is similar to the one shown in Figure 8. The gain of the discriminator is dependent upon the magnitude of the input voltage. For this reason it is desired that the amplitude of the input signal be large. A large input signal is also required to drive the diodes of the discriminator far into the conduction region. This is necessary to prevent a dead zone near the resonant frequency of the discriminator due to poor conduction of the diodes.

To increase the amplitude of the signal, an amplifier preceded the discriminator. A Fairchild $\mu$709-C integrated circuit operational amplifier was used for this purpose. It followed a transistor amplifier which buffered the tuned circuit of the oscillator. The circuit configuration of this amplifier is shown in Figure 11(a). The output swing of the operational amplifier was 22 volts.

To increase the total loop gain, a dc amplifier

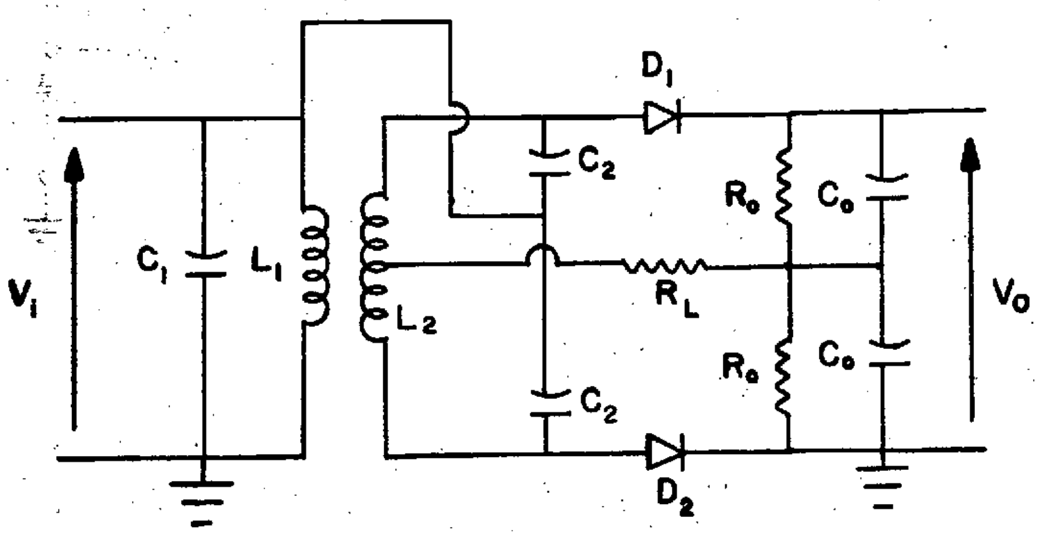



FIGURE 10.--A modified Foster-Seeley discriminator

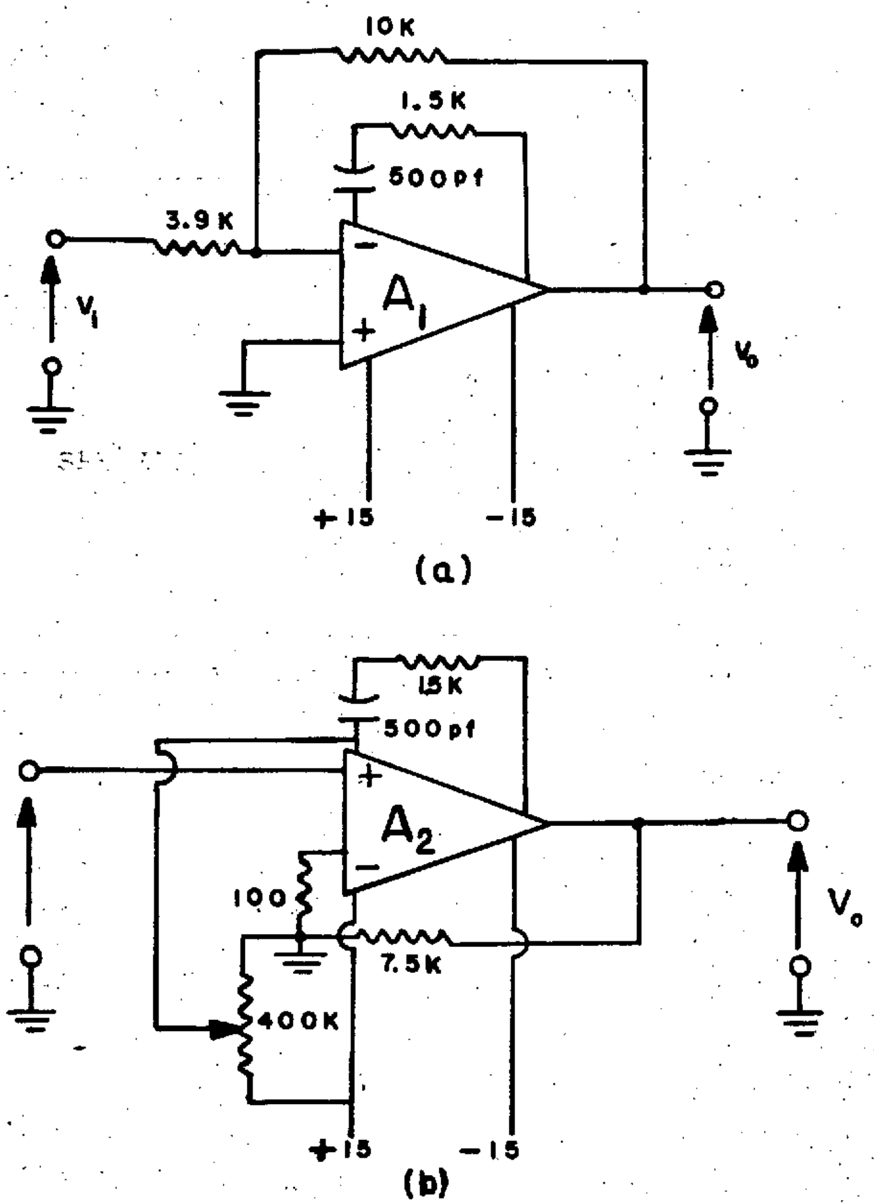



FIGURE 11.--(a) Inverting operational amplifier with frequency compensation; (b) Non-inverting dc amplifier with frequency compensation and offset compensation

followed the discriminator. A Fairchild $\mu$709-C high performance operational amplifier with a high gain and a low offset voltage was used. The circuit configuration of the Fairchild $\mu$709-C dc amplifier is shown in Figure 11(b), and the pertinent specifications are given in Table I.

TABLE I

SPECIFICATIONS OF THE $\mu$-709-C OPERATIONAL AMPLIFIER*

| Quantity | Value |
|---|---|
| supply voltage | $\pm$ 18 volts |
| input voltage | $\pm$ 10 volts |
| operating temperature range | 0°C to 70°C |
| input offset voltage | 10 nv typical |
| input offset current | 750 na typical |
| transient response | |
| rise time | 1.0 $\mu$s (max) |
| overshoot | 30% (max) |

* Fairchild semiconductor

Despite the exceptional characteristics of this operational amplifier, it places certain restrictions on the automatic frequency control loop. The two most important restrictions are due to its temperature operating range and the input offset voltage.

A schematic diagram of the automatic frequency control system is shown in Figure 12. In this case the type low-pass filter used is dictated by the configuration of the oscillator circuit. The filter consists of $R_f$, $L_f$, and $C_f$, shown in Figure 12. $C_f$ is necessary to isolate, dc wise, the varactor anode from ground potential. Two things must be remembered when choosing a value for this capacitor. Since it is part of the filter, it affects the response of the system to an error. This capacitor also determines the tuning range to some extent. For the tuning range to be a maximum, this capacitance must be much greater than that of the varactor. The inductor, $L_f$, is also required because of the circuit configuration. It isolates the rf of the oscillator from the output of the operational amplifier. The coil must present a high impedance at the frequency of the oscillator (440 kHz).

There are many possible filter parameters which will give desirable performance. These parameters depend upon the criteria of performance and the nature of modulation of the oscillator. The procedure is as follows:

(1) determine the gain required for the stability desired; (2) using the equations from the preceding section, determine values for $L_f$ and $C_f$ to give the

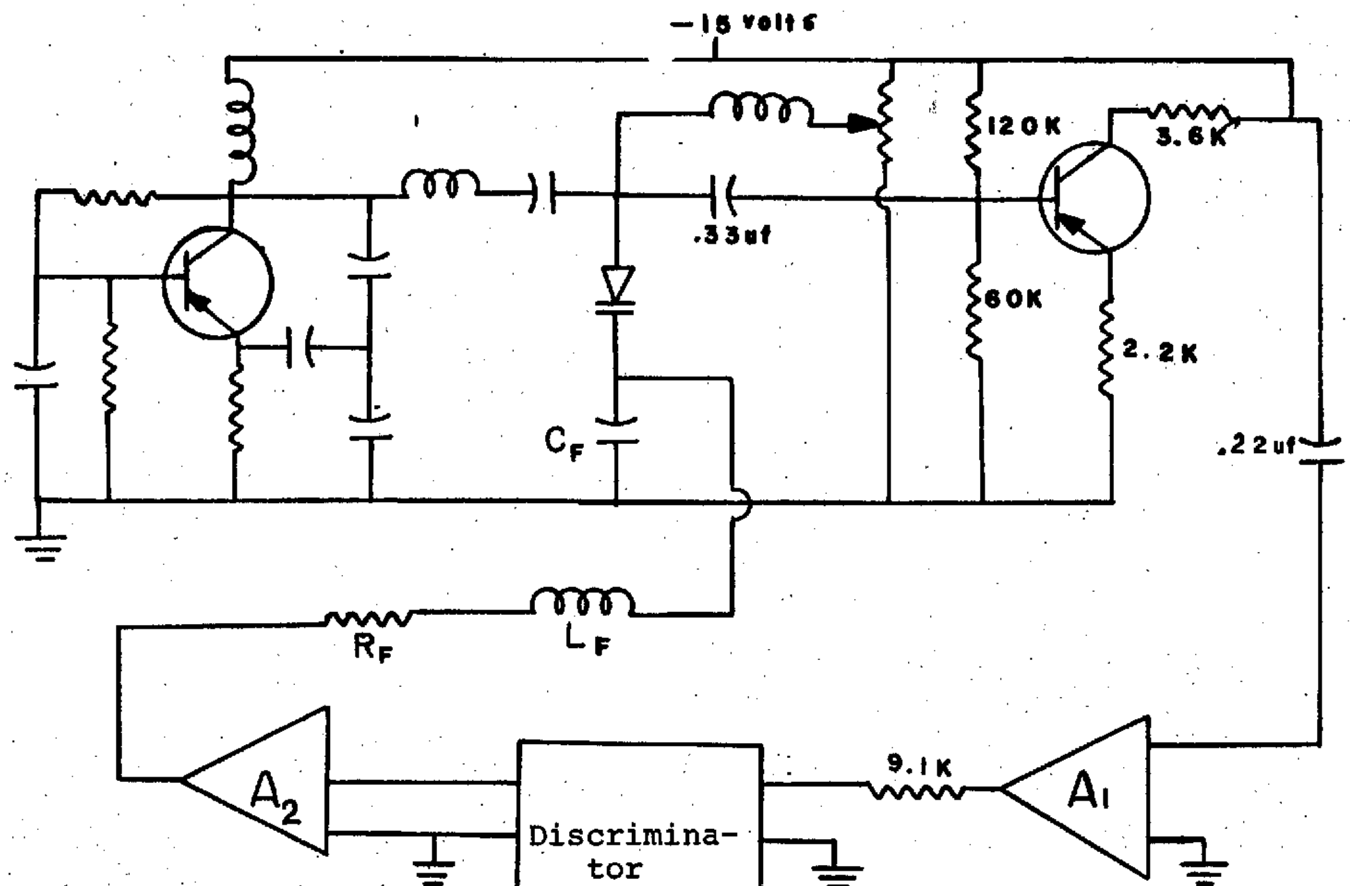



FIGURE 12.--A schematic diagram of the AFC system

wanted $\omega_n$;(3) choose $R_f$ to give optimum damping for the loop.

In general, it is desirable that operation of the circuit should not affect the amplitude of the oscillations but should act to vary the frequency only. For better sensitivity, a large change in frequency should be produced by a small change in the control bias of the oscillator. This relationship should hold over a sufficient frequency interval to take care of the greatest amount of unwanted mistuning or oscillator drift that will normally be met.

## IV. EXPERIMENTAL RESULTS

Due to the unavailability of a diode with sufficient capacitance, the varactor used in the oscillator was a parallel combination of five individual diodes. The voltage-capacitance curve for the combined diodes is shown in Figure 13.

The varactor was biased at $V_o = -4$ volts with a quiescent capacitance, $C_o$, of 113 Pf. The limit which the oscillator places on the capture and hold-in ranges can be observed from the curve in Figure 13. The total voltage applied to the varactor, which is the sum of the bias voltage, the correction voltage and the rf voltage, must not drive the voltage across the diode past the breakdown voltage, $V_B$. It can be seen from the curve in Figure 13 that the lower limit on the capacitance, $C_{min}$, is 65 pf. If it is assumed that zero volts is the highest applied voltage, $C_{max}$ has the value 217 pf. This ratio of $C_{max}$ to $C_{min}$ will give a frequency tuning range of about 200 kHz. A plot of the change in frequency versus the error voltage is shown in Figure 14. The gain of the oscillator in Hertz per volt is found by determining the slope of the line in Figure 14. In the region around

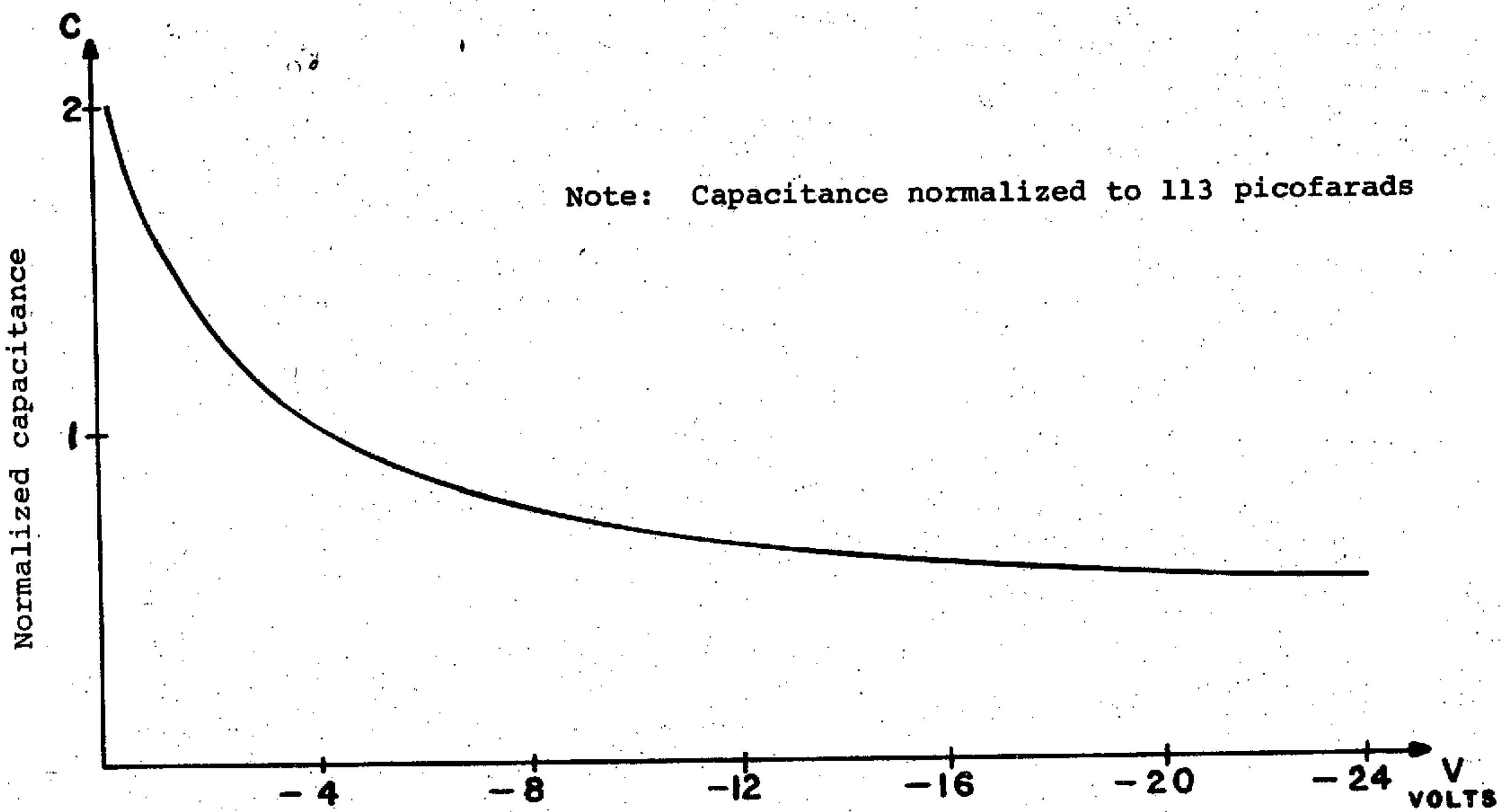
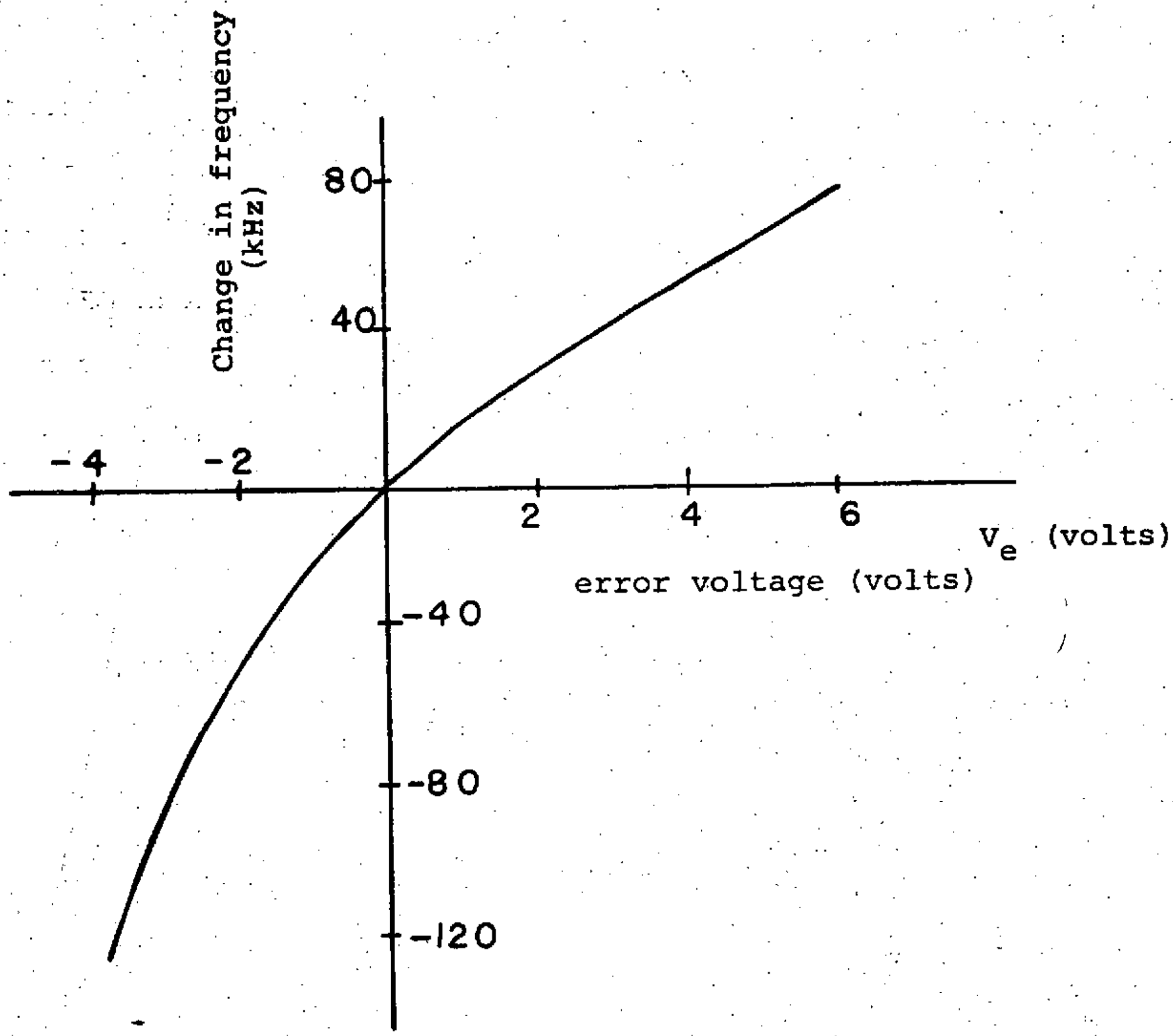



FIGURE 13.--Normalized capacitance versus the bias voltage of the varactor diode




Note: $V_o$ = -4 volts

$f_o$ = 440 kHz

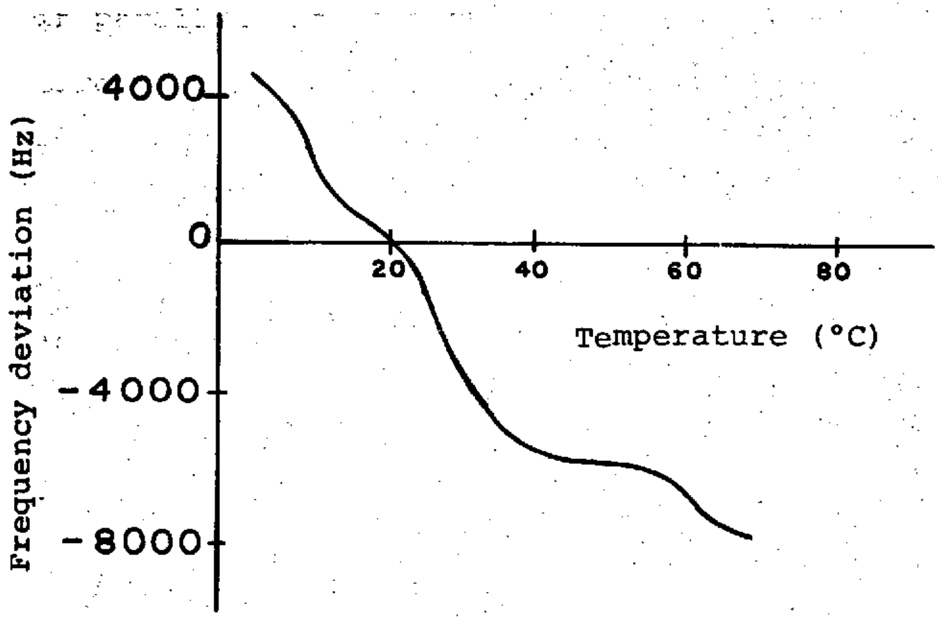
FIGURE 14.--The change in frequency versus the error voltage applied to the varactor of the oscillator

the bias point the curve can be approximated by a straight line with a slope of 17.5 KHz per volt. If the error voltage does not exceed 1.5 volts, the assumption that the oscillator characteristic curve is approximately linear is valid.

Environmental temperature is probably the most important variable affecting the stability of the oscillator, especially if it is to be used over a wide range of temperatures. To determine the effect of a change in temperature on the frequency of oscillation, the oscillator was placed in an environmental chamber in which the temperature was varied from 5°C to 70°C. The automatic frequency loop was not closed, and the oscillator was allowed to take on its free running frequency. The results of this test are shown in Figure 15.

The curve in Figure 15, while certainly not linear, has a predictable form. As the temperature increases the frequency decreases, which perhaps could be expected since the LC parameters determining the frequency of oscillation are related to temperature [15]. The overall stability of the oscillator over a 70° degree temperature range is about 1.5%.

The stability of the center frequency of the discriminator is of particular interest. It has been shown in the analysis that the closed loop stability is dependent



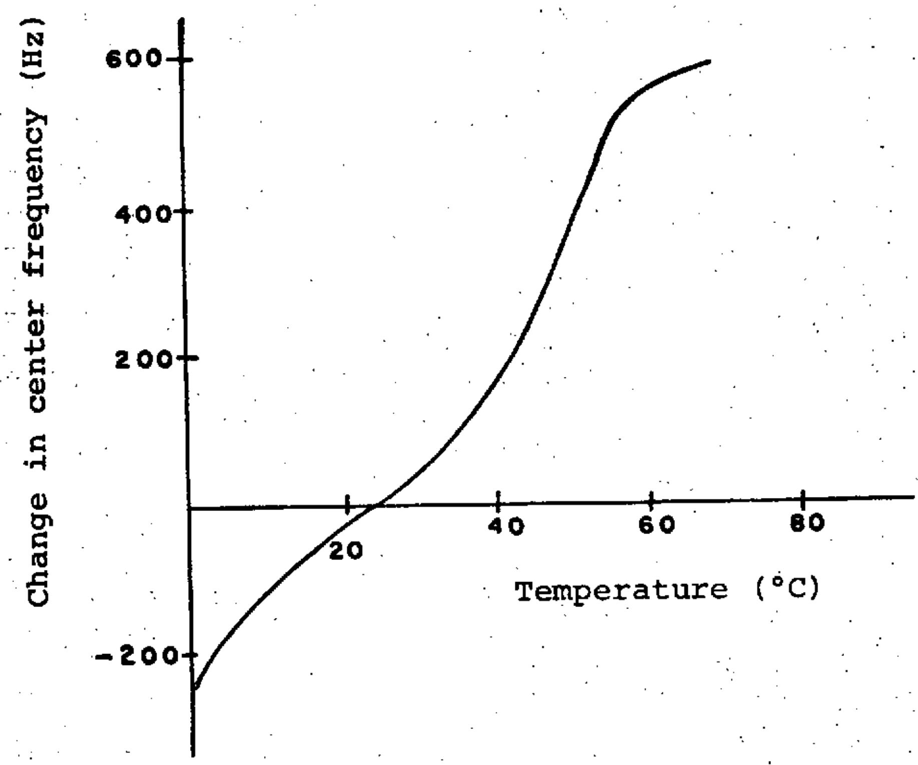

Note: The reference temperature is 20°C.

FIGURE 15.--The frequency deviation of the oscillator versus temperature of the oscillator

on the stability of the center frequency of the discriminator. The change in the center frequency of the discriminator is plotted against temperature in Figure 16. The stability of this discriminator over a 70° temperature range is about .1%. The slope of the curve in Figure 16 is not fixed. A small capacitor placed in parallel with one of the capacitors in the secondary circuit can either increase or decrease the slope of the curve, depending upon whether the capacitors have temperature coefficients of capacitance of the same or opposite signs. The significance of this is seen by looking at (8), which is rewritten below.

$$\delta_o = \delta_d + \frac{\delta_a - \delta_d}{1+K} \tag{8}$$

The output stability will be less than or greater than the stability of the discriminator center frequency if the oscillator stability and the discriminator center frequency stability are of the same or opposite signs respectively. While some variables cause random changes in the output fxequency, it is shown in Figure 15 that variations due to temperature are predictable. Capacitive compensation can be used to make the slope of the discriminator frequency versus temperature curve to be of opposite sign than that of the oscillator frequency versus temperature curve. This tends to mkae the output frequency more




Note: The reference frequency is 440 kHz at 22°C

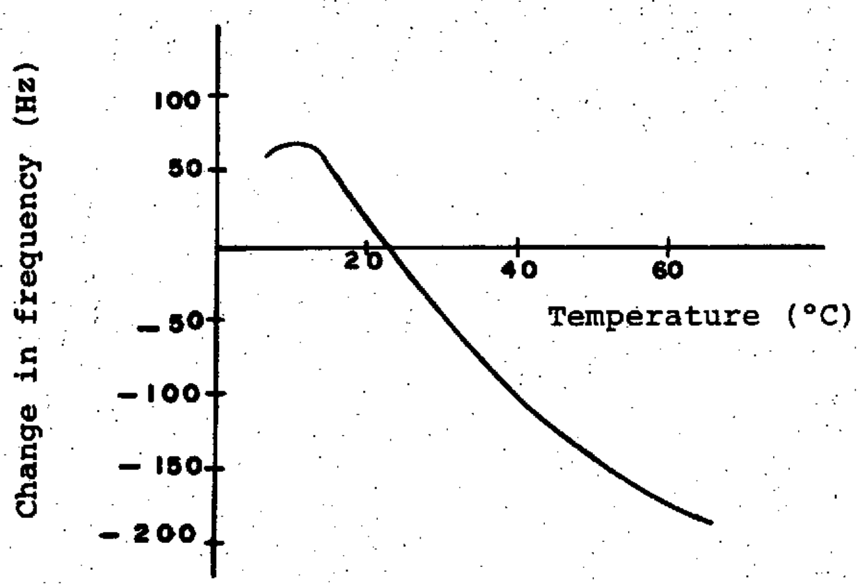
FIGURE 16.--The change in center frequency of discriminator versus temperature of discriminator

accurate than the center frequency of the discriminator. The evidence of this is shown in Figure 17.

The stability of the entire loop over a 70° temperature variation is about .05%, compared with 1.5% for the oscillator and about .1% for the discriminator. When the hold-in range and the capture range are discussed, the characteristic curve of the discriminator can be helpful. For a constant temperature the output voltage was measured as the input frequency was varied. The results are shown in Figure 18. The bandwidth of this discriminator was approximately 20 kHz. The curve is fairly linear between 430 kHz and 450 kHz. The gain constant of the discriminator, $K_1$, is the slope of the curve. For frequencies near the resonant frequency the curve can be approximated by a straight line with slope of .52 volts per kHz. The range over which the output can be detected exceeds 200 kHz.

A plot of the closed-loop compensated frequency versus the uncompensated frequency of the open loop is shown in Figure 19. The hold-in range and the capture range can be determined from the curve in Figure 19. It was found for the model constructed that neither range was determined at the lower limit by the discriminator.

At a frequency of 360 kHz, where the conditions necessary for oscillation failed to be satisfied by the oscillator,




Note: The reference frequency is 440 kHz at 22°C.

FIGURE 17.--The change in closed-loop oscillator frequency versus temperature

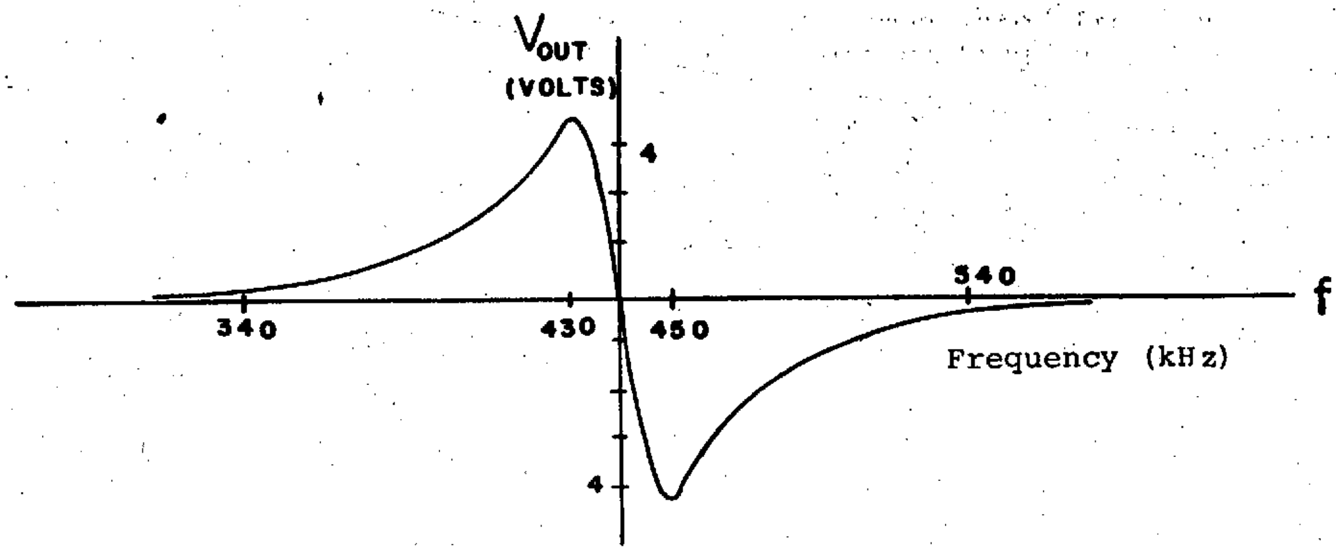



FIGURE 18.--The characteristic curve of the constructed discriminator

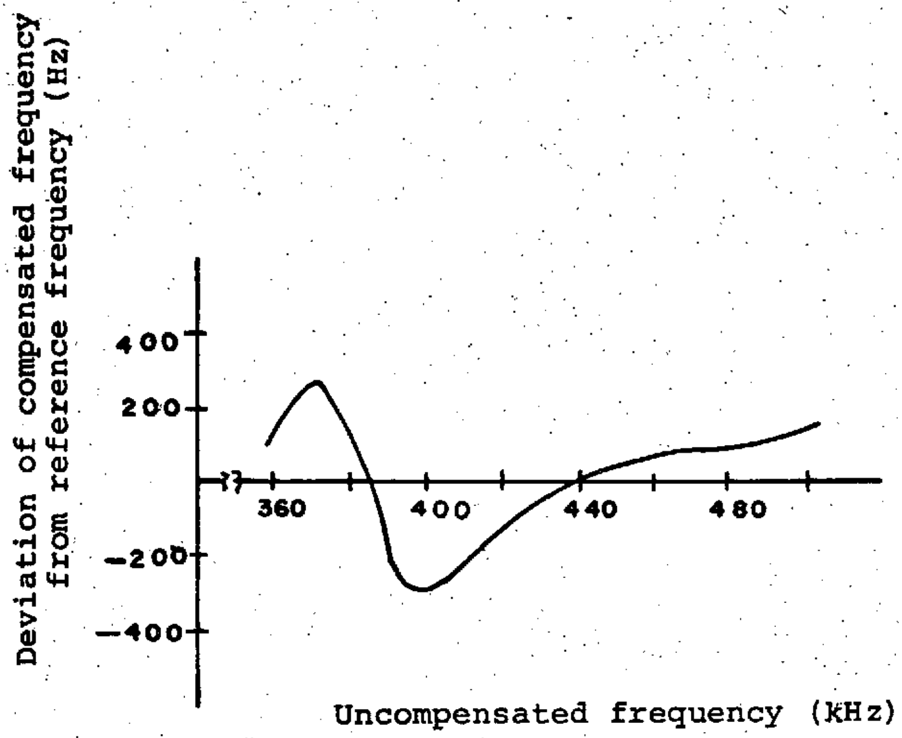



FIGURE 19.--The difference of the reference frequency and the closed-loop compensated frequency versus the open-loop uncompensated frequency for a constant temperature

the AFC loop was still functioning. The frequency of oscillation was changed by detuning the coil in the tuned circuit of the oscillator. This range of inductance of the coil prevented detuning of the output to frequencies over 545 kHz. This frequency of 545 kHz was still within the hold-in range and the pull-in range of the automatic frequency control loop.

## V. CONCLUSIONS

The principles of operation of a simple automatic frequency control system utilizing a Colpitts oscillator, a varactor modulator, and a discriminator were discussed and a mathematical analysis of this system was presented. From this analysis it was shown that the frequency stability of the controlled oscillator is dependent upon the stability of the center frequency of the discriminator, the open-loop gain of the system, and the open-loop frequency of the oscillator.

Design considerations, drawn from this analysis, were presented, and the physical limitations encountered in the design of a system were discussed.

A model of the automatic frequency control system was designed and constructed. The discriminator was temperature compensated to improve its center frequency stability. It was shown, from the several tests performed on the model, that the accuracy of the output frequency can exceed that of the center, or reference, frequency of the discriminator. It was shown, by using a discriminator with a center frequency stability of 0.1 percent and an oscillator with a frequency stability of 1.5 percent over a temperature range of 5°C to 70°C, that the overall stability can exceed .05 percent.

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